Announcements

- Regrade requests are due a week after we open the grades. ~~Today for the prelim~~ Was closed, almost set with responses

- Tuesday OH Eva Tardos: 10:30-11:30. would 2-3 be better?

- Hw3 is graded
- Hw5 is posted on Gradescope/canvas

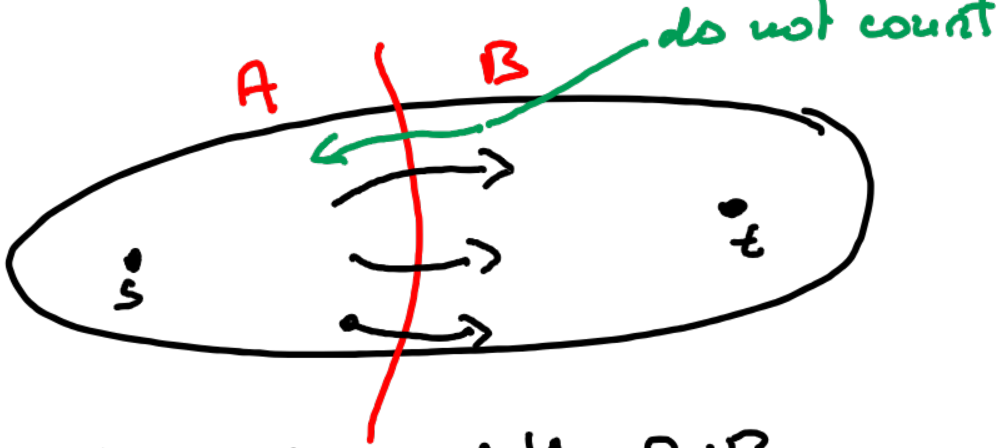
today: application of cuts

Monday start of Chapter 8

Today min cut applications.

The min-cut problem

Input $G=(V,E)$ directed
$s,t \in V$, capacities $c_e \geq 0$ for $e \in E$



Find partition of $V = A \cup B$
$s \in A,\ t \in B$
& min capacity $(A,B) = \sum_{e \text{ from } A \text{ to } B} c_e$

Ford-Fulkerson
max flow value = min cut capacity

& algorithm finds a min cut

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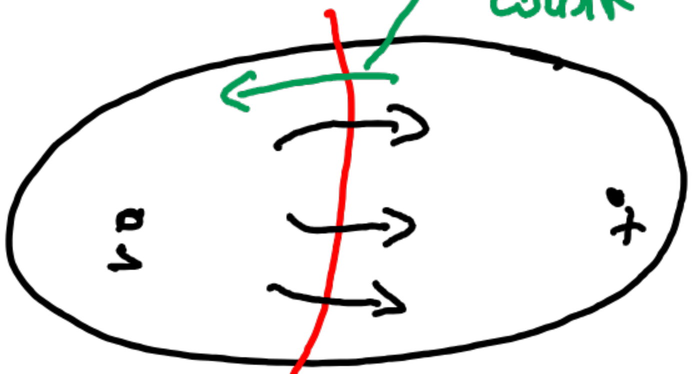
What is the min-cut value on the network below

- A. 6
- B. 4
- C. 7

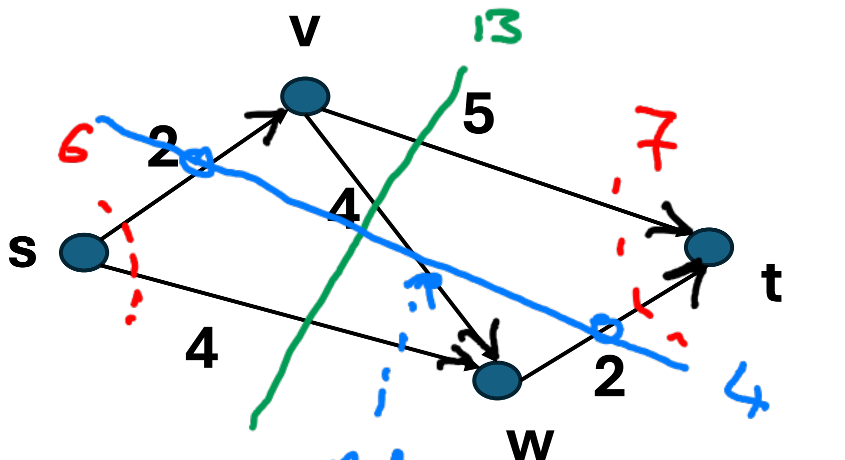
too high

does not count



cut value = sum capacities $A \to B$

network C



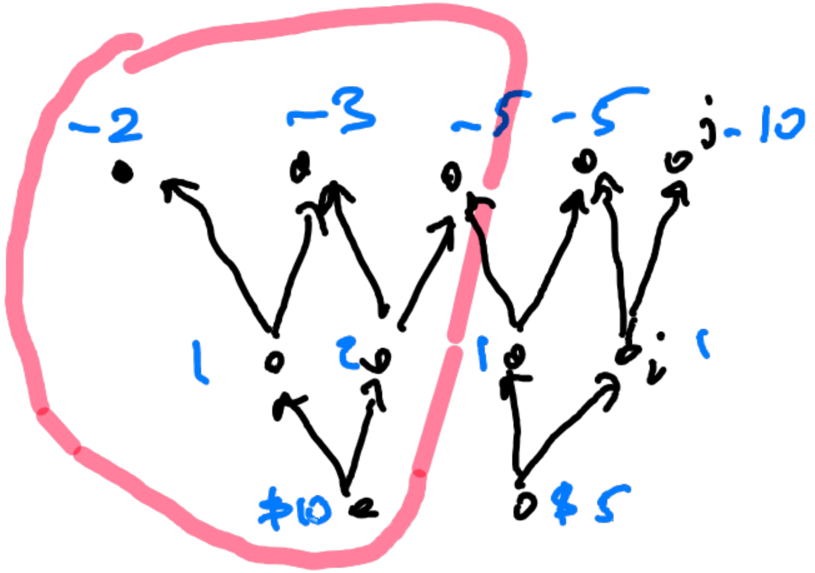
edge going $B \to A$ does not count

The project selection problem

project to maybe do

p_j = value for doing project j
may be negative

some project i has requirements: need to also select some other projects



projects = nodes
edge (i,j) means i requires j also

Problem: find subset of projects satisfying requirements & max total value

Open pit mining

total 3

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What is the value of the optimal set of projects?

A. 30

B. -7

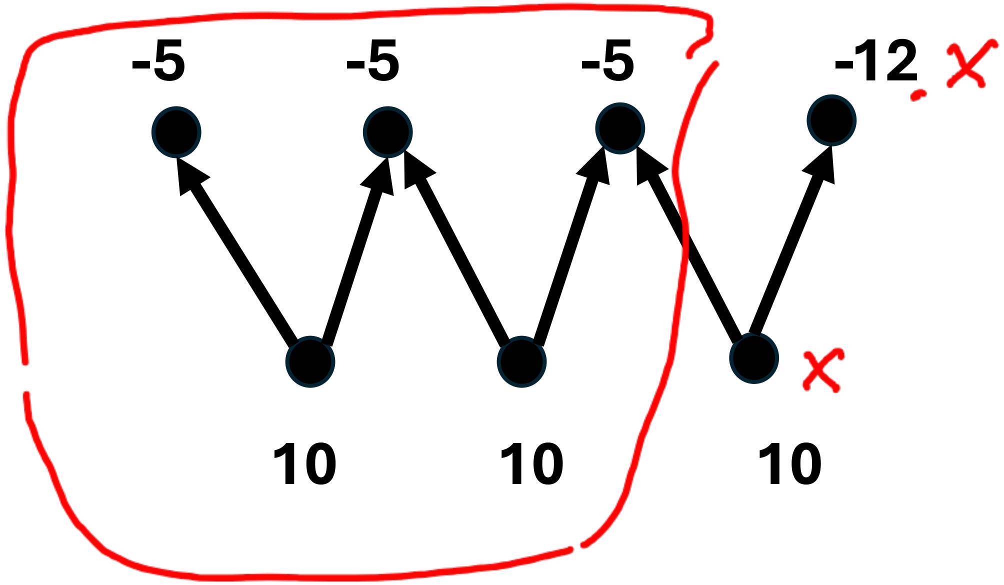
C. 5

D. 0

$(i,j) \in E$

i requires j to be taken

goal: max value



Project selection as a cut problem

Select cut in graph (A, B)

no edges from $A \to B$

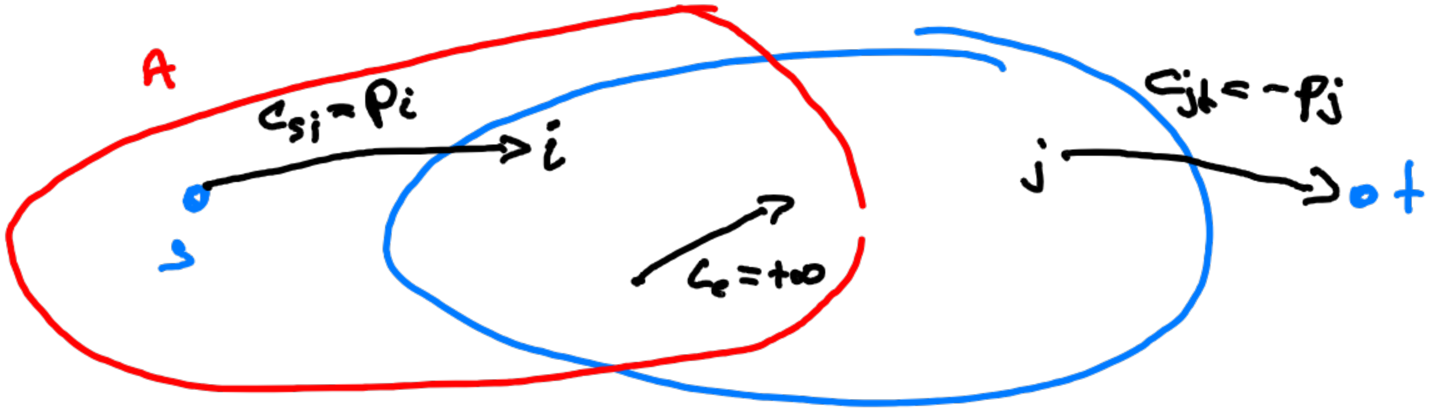
Idea 1: $c_{ij} \approx +\infty \implies$ min cut never contain them

comment $c_e = +\infty$ OK in algorithm

also ok $c_e = \sum_j |p_j|$

no min cut will have $A \to B$ edge of this form

Project selection solvable by Ford-Fulkerson



add s + t

edges (s,i) if $p_i > 0$

(j,t) if $p_j < 0$

Claim: min cut (A,B) in this network

$\Rightarrow$ $A \setminus s$ = optimal set of projects

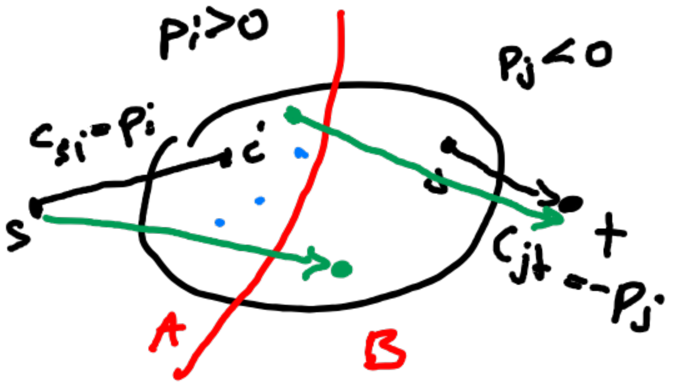
Claim 1: cut with finite capacity

1-to-1 correspondence with valid project sets

if & only if $i \in A$ & $(i,j) \in E$ $\Rightarrow j \in A$

need: min cut capacity ~ max project value

Proving the Algorithm Correct



value projects $= \sum_{j\in A} p_j$

capacity $(A,B) = \sum_{j\in B:\, p_j>0} p_j - \sum_{i\in A:\, p_i<0} p_i$

$$= \sum_{j\in B} p_j - \sum_{j\in B:\, p_j<0} p_j - \sum_{i\in A\, p_i<0} p_i = \sum_{j\in B} p_j - \sum_{i:\, p_i<0} p_i$$

fixed does not depend on cut

$$= \boxed{\sum_{j:\, p_j>0} p_j} - \sum_{j\in A} p_j$$

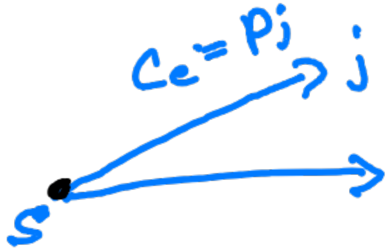
fixed does not depend on cut

Note: error in last line fixed after class

max value project set

= min capacity cut

Running time:



if $p_j > 0$

Ford-Fulkerson OK is
p_j values are small integers

better algorithm depends on
choice of path

$$C = \sum_{e=(s,i)\in E} c_e$$

iterations $\leq C+1$ if
c_e integer all e

One iteration $O(m+n)$
m = # edges
n = # nodes

time $O(C(n+m))$